

# **FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012** **Match 3 Team Round**

1.) Compute  $FCML_{30} + MATH_{30}$ . Write your answer in base 30.

2.) Six grams of the color Aqua is made from mixing 5 grams of blue with 1 gram of green. Three grams of the color Sea Gold is made from mixing 2 grams of blue with 1 gram of green. Five grams of the color Turquoise is made from mixing 4 grams of blue with 1 gram of green. Dave had just enough materials to make 40 grams of Aqua and 15 grams of Sea Gold. How many grams of Turquoise could he have made, discarding any leftover materials?

3.) There are two regular  $n$ -gons such that the interior angle of the regular  $n$ -gon is  $(10n)^\circ$  more than its exterior angle. Find the positive difference between the number of diagonals in the two  $n$ -gons.

4.) If  $f(x+10) = 2x+6$ , then find all values of  $x$  such that  $f(x) = f^{-1}(x)$ .

5.) If  $a = \log_5 4$  and  $b = \log_8 27$ , then evaluate  $(5^a 2^b)^2$ .

6.) If  $M = \begin{bmatrix} a & 0 & b \\ b & -b & a \\ 0 & b & b \end{bmatrix}$  and  $N = \frac{1}{5} \cdot \begin{bmatrix} c & -b & -b \\ b & -a & c \\ -b & a & a \end{bmatrix}$  are inverses, and  $P = \begin{bmatrix} a & b \\ b & 0 \end{bmatrix}$  and

$\bar{O} = \frac{1}{2} \cdot \begin{bmatrix} b & a \\ -b & 0 \end{bmatrix}$ , then find all possible ordered triplets  $(a, b, c)$ .

TEAM

1) 17NM8<sub>30</sub>

2)  $\frac{325}{6}$  grams

3) 45

4)  $X = 14$

5) 144

6)  $(1, 2, 3)$  or  $(-1, -2, -3)$

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 3 Round 1

Arithmetic: Bases and Scientific Notation

1.)  $1.5 \times 10^{15}$

2.)  $24200_5$

3.)  $1031121_4$

Please Note The symbol  $12_B$  represents a 2-digit number in base B.

1.) Express  $\frac{(12 \cdot 10^{18})(20 \cdot 10^9)(15 \cdot 10^{11})}{(24 \cdot 10^{25})}$  in scientific notation.

2.) Simplify  $(30 \cdot 10^6)(20 \cdot 10^{14})(30 \cdot 10^{-21})$  and express your answer as a base 5 numeral.

3.) Calculate  $1248_{16} + 421_8$ . Express your answer as a base 4 numeral.

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2010-2011

Match 3 Round 2  
Algebra I: Word Problems

$$1.) \quad \begin{array}{r} 1000 \\ 9 \overline{) 1} \end{array}$$

$$2.) \quad K_1 = 64 \text{ or } 64$$

$$3.) \quad \begin{array}{r} 24 \\ 13 \overline{) 24} \end{array}$$

1.) Mixture A contains 100% juice. Mixture B contains 80% juice and 20% water. If there are 100 L of each mixture available, how many liters of a mixture containing 98% juice and 2% water is it possible to make?

2.) Adding their scores, Ellen and Kelly scored a total of 114 points on the first assignment. Ellen scored 13 more points on her second assignment than on her first assignment. Kelly scored 11 fewer points on her second assignment than on her first assignment. If Kelly scored four more points on her two assignments than Ellen scored on her two assignments, then find how many points Kelly scored on her first assignment.

3.) Working together, Al and Bob can complete a job in 2 days. It takes Bob and Charlie three days working together to complete the same job. Working together, it takes Al and Charlie 4 days to complete the same job. How many days will it take to complete the same job if Al, Bob, and Charlie all work together?

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 3 Round 3  
Geometry: Polygons

1.) 3, 4, 5, 6, 7

2.) 98

3.)  $2\sqrt{2}$   
3

1.) List all possible values for  $n$  such that a regular  $n$ -gon has interior angles that are less than three times its exterior angles.

2.) A regular polygon with 14 diagonals shares a side with a regular polygon with 9 sides. If the shared side has a length of 7, then what is the perimeter of the polygon which is formed when the shared side is removed?

3.) Regular polygon **T** has 3 sides, regular polygon **S** has four sides, and regular polygon **H** has six sides. The ratio of the apothem of **T** to the apothem of **H** is the same as the ratio of the perimeter of **H** to the perimeter of **S**. If the area of **T** is the same as the area of **H**, then find the ratio of the apothem of **H** to the apothem of **S**.

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 3 Round 4

Algebra II: Functions and Inverses

- 1.)  $\frac{21}{x+5}$
- 2.)  $\frac{x-1}{x+5}$
- 3.)  $\left( \frac{-\sqrt{15}}{2}, \frac{\sqrt{15}}{2} \right)$

1.) The function  $g$  has the property that  $g(x) = 1 + g(x-1)$  for all real  $x$ , and  $g(0) = 1$ . The function  $f$  is defined by  $f(x) = x + g(x)$ . Evaluate  $f(10)$ .

2.) Let  $f(x) = x^2 - 4$ ,  $g(x) = x + 3$ , and  $h(x) = \frac{f(g(x))}{g(f(x))}$  for all  $x > 1$ . Simplify  $h(x)$ .

3.) If the function  $f$  is defined by  $f(x) = \frac{1}{\sqrt{4-x^2}}$ , then what is the domain of the function  $f \circ f$ ?

[Explanation of notation: For any function  $f$ , the function  $f \circ f$  is defined by  $f \circ f(x) = f(f(x))$ , for all values of  $x$  for which  $f(f(x))$  is real.]

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 3 Round 5  
Advanced Topics: Exponent and Log  
Expressions and Equations

- 1.)  $2\frac{-3}{3}$   
2.)  $\frac{1}{9}, 3$   
3.)  $8/0$

1.) Evaluate  $\frac{\log 8}{\log(\frac{1}{4})}$ .

2.) Solve for x:  $1 + \log_3 x = \log_x 9$

3.) Given that  $2^{5x+2y} = 2^{3x} + 2^{3x+3}$  and  $3^{-3x} = 3^{3y} - 3^{-3x-2}$ , evaluate  $(12)^{3x+3y}$

KCY

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2010-2011

Match 3 Round 6  
Discrete Math: Matrices

1.)  $\frac{-37}{60}$

2.)  $-8$

3.)  $\begin{bmatrix} 5/4 & 3 \\ 1 & 5/2 \end{bmatrix}$

1.) If  $\begin{bmatrix} 10x & 1 \\ -4 & 4z \end{bmatrix} \cdot \begin{bmatrix} 30y & 5 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ -\frac{5}{36} & 9 \end{bmatrix}$ , then find  $x + y + z$ .

2.) Find the determinant of  $\begin{bmatrix} 187 & 189 \\ 191 & 193 \end{bmatrix}$ .

3.) If  $A = \begin{bmatrix} -4 & 1 \\ 4 & -2 \end{bmatrix}$  and  $A \cdot X \cdot A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ , then find  $X$ .



# **FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012**

Match 3 Round 1

Arithmetic: Bases and Scientific Notation

1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.) \_\_\_\_\_

*Please Note The symbol  $12_B$  represents a 2-digit number in base B.*

1.) Express  $\frac{(12 \cdot 10^{18})(20 \cdot 10^9)(15 \cdot 10^{11})}{(24 \cdot 10^{25})}$  in scientific notation.

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Match 3 Round 2  
Algebra I: Word Problems

1.) \_\_\_\_\_

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**FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012**

Match 3 Round 3  
Geometry: Polygons

- 1.) List all possible values for  $n$  such that a regular  $n$ -gon has interior angles that are less than three times its exterior angles.
- 1.) \_\_\_\_\_
- 2.) \_\_\_\_\_
- 3.) \_\_\_\_\_

2.) A regular polygon with 14 diagonals shares a side with a regular polygon with 9 sides. If the shared side has a length of 7, then what is the perimeter of the polygon which is formed when the shared side is removed?

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# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 3 Round 4  
Algebra II: Functions and Inverses

1.) \_\_\_\_\_

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3.) \_\_\_\_\_

1.) The function  $g$  has the property that  $g(x) = 1 + g(x-1)$  for all real  $x$ , and  $g(0) = 1$ . The function  $f$  is defined by  $f(x) = x + g(x)$ . Evaluate  $f(10)$ .

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# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2010-2011

Match 3 Round 6  
Discrete Math: Matrices

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Match 3 Round 6  
Discrete Math: Matrices

- 1.) \_\_\_\_\_
- 2.) \_\_\_\_\_
- 3.) \_\_\_\_\_

1.) If  $\begin{bmatrix} 10x & 1 \\ -4 & 4z \end{bmatrix} \cdot \begin{bmatrix} 6 & 5 \\ 30y & 9 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ -\frac{5}{36} & 9 \end{bmatrix}$ , then find  $x + y + z$ .

2.) Find the determinant of  $\begin{bmatrix} 187 & 189 \\ 191 & 193 \end{bmatrix}$ .

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